# Thermal Conductivity of Misaligned Short-Fiber-Reinforced Polymer Composites

**Shao-Yun Fu,**[1–3] **Yiu-Wing Mai**[2,3]

[1]*Technical Institute of Physics and Chemistry, Chinese Academy of Sciences, Beijing 100080, China*
[2]*Department of Manufacturing Engineering and Engineering Management (MEEM), City University of Hong Kong, 83 Tat Chee Avenue, Kowloon, Hong Kong, China*
[3]*Centre for Advanced Materials & Technology (CAMT), School of Aerospace, Mechanical & Mechatronic Engineering J07, The University of Sydney, NSW 2006, Australia*

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**ABSTRACT:** A fiber length distribution (FLD) and a fiber orientation distribution (FOD) usually exist in injection-molded short-fiber-reinforced polymer (SFRP) composites. In this article, the thermal conductivity of SFRP composites is studied in detail, taking into account the effects of the FLD and the FOD. The effect of fiber volume fraction on the composite thermal conductivity is also investigated. It is shown that the thermal conductivity of SFRP composites increases almost linearly with fiber volume fraction; it also increases with mean fiber length (or mean aspect ratio) but decreases with mean fiber orientation angle with respect to the measured direction. The latter two effects depend highly on the thermal conductivity of short fibers. The effects of mode fiber length and mode fiber orientation angle on the thermal conductivity of short (carbon) fiber reinforced polymer composites have also been studied. The composite thermal conductivity increases marginally with the increase of mode fiber length but lightly with the increase of mode fiber orientation angle. Moreover, comparison of the present model is made with existing theories in predicting the thermal conductivity of SFRP composites. Finally, the present model is applied to published experimental results and the agreement is found to be satisfactory. © 2003 Wiley Periodicals, Inc. J Appl Polym Sci 88: 1497–1505, 2003

**Key words:** thermal conductivity; composites; injection molding; fiber length distribution; fiber orientation distribution

## INTRODUCTION

Short-fiber-reinforced polymer (SFRP) composites have many applications as a class of structural materials because of their ease of fabrication, relatively low cost ,and superior mechanical properties to those of relevant polymer resins. Extrusion compounding and injection-molding techniques are often employed to make SFRP composites.[1–12] Short fibers are often misaligned due to processing. Fibers are also damaged due to processing. Hence, in the final composites, there exist a fiber length distribution (FLD) and a fiber orientation distribution (FOD). Studies on the mechanical properties of SFRP composites have shown that both the FLD and the FOD play very important roles in determining the mechanical properties.[13–19] SFRP composites are also attractive materials for electronic packaging applications where the combination of reinforcement with high thermal conductivity embedded in a resin matrix with low thermal conductivity is desirable to maintain a low temperature environment for thermally sensitive electronic packaging components. A number of analytical models have been proposed to predict the thermal conductivity of short fiber composites.[20–26] They are, however, focused on either aligned short fiber composites[20,21] or completely random short fiber composites[22,23] or short fiber composites with fibers of a constant fiber length.[24–26] FLD always exists in injection-molded SFRP composites. However, all the above models did not study the effect of FLD on the thermal conductivity of short fiber composites. It will be shown that the FLD plays an important role in determining the thermal conductivity. Moreover, due to partial fiber alignment, SFRP composites could show anisotropy in their properties. The anisotropies of the elastic modulus and strength of SFRP composites have been studied previously,[13,14] but the anisotropy of the thermal conductivity of SFRP composites has not been investigated in detail yet by considering the effects of the FLD and the FOD.

Thermal conductivity is a bulk property, analogous to modulus. Moreover, it is well accepted that a mathematical analogy exists between thermal conduction and elasticity of fiber composites. The elastic modulus of SFRP composites has been successfully predicted using the laminate analogy approach (LAA).[15] Therefore, in the present work, the LAA is also employed to derive an expression for the thermal conductivity of

*Correspondence to:* S.-Y. Fu (syfu@cl.cryo.ac.cn).

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SFRP composites, taking into consideration the effects of the fiber volume fraction, the FLD, and the FOD. Comparison of the present model for the prediction of the thermal conductivity of SFRP composites is made with other existing theories.[24,25] Moreover, the present theory is applied to published experimental results.[27,28] The anisotropy of the composite thermal conductivity will be reported elsewhere.

## THEORY

### FLD and FOD

The FLD density function, $f(L)$, can be given as follows[13–18,29]

$$f(L) = abL^{b-1}\exp(-aL^b) \quad \text{for } L > 0 \tag{1}$$

where $a$ and $b$ are size and shape parameters, respectively, for determining the size and shape of FLD curves. The mean fiber length is given by

$$L_{\text{mean}} = \int_{L_{\min}}^{L_{\max}} Lf(L)dL = a^{-1/b}\Gamma(1/b + 1) \tag{2}$$

where $\Gamma(x)$ is the gamma function. The mode fiber length is given by

$$L_{\text{mod}} = [1/a - 1/(ab)]^{1/b} \tag{3}$$

The mean fiber length ($L_{\text{mean}}$ in mm) and the mode fiber length ($L_{\text{mod}}$ in mm) can be measured experimentally. Then, the parameters $a$ and $b$ can be obtained from the following equations, respectively:

$$(1 - 1/b)^{1/b}/(1/b + 1) = L_{\text{mod}}/L_{\text{mean}} \tag{4}$$

$$a = (1 - 1/b)/L_{\text{mod}}^b \tag{5}$$

Fiber orientation can be described by a pair of angles $(\theta,\phi)$ as shown in Figure 1. The FOD density function, $g(\theta)$, can be defined by[13–18]

$$g(\theta) = \frac{(\sin\theta)^{2p-1}(\cos\theta)^{2q-1})/}{\int_{\theta_{\min}}^{\theta_{\max}} (\sin\theta)^{2p-1}(\cos\theta)^{2q-1}d\theta} \tag{6}$$

where $p$ and $q$ are the shape parameters that can be used to determine the shape of the FOD curve, and $0 \le \theta_{\min} \le \theta \le \theta_{\max} \le \pi/2$. The mean fiber orientation angle ($\theta_{\text{mean}}$) is obtained:

$$\theta_{\text{mean}} = \int_{\theta_{\min}}^{\theta_{\max}} \theta g(\theta)d\theta \tag{7}$$

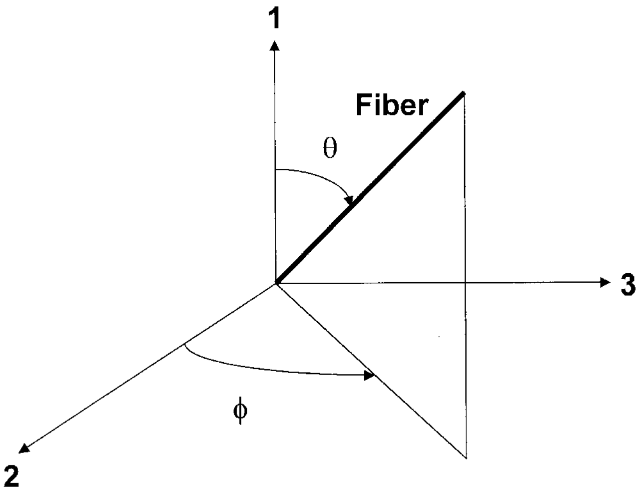



**Figure 1** Definitions of spatial fiber orientation angles $\theta$ and $\phi$.

The mode fiber orientation angle is given by

$$\theta_{\text{mod}} = \arctan\{[2p - 1)/(2q - 1)]^{1/2}\} \tag{8}$$

The FOD density function, $g(\phi)$, can also be defined similar to that of $g(\theta)$.[13]

### Thermal conductivity of SFRP composites

The LAA will be used here to evaluate the thermal conductivity of SFRP composites. In the LAA, the SFRP composites can be simulated as a sequence of a stack of various laminae with different fiber orientations and different fiber lengths. The successive development of the laminated plate model of a three-dimensionally (3D) misaligned SFRP is shown in Figure 2. The SFRP composite with a 3D spatial FOD function $g(\theta,\phi)$ [$= g(\theta)g(\phi)/\sin\theta$] having fiber ends in the three visible planes [see Fig. 2(a)] is first replaced by an SFRP with the same $g(\theta)$ but $\phi = 0$, having no fiber ends in the 1–2 plane or no fibers in the out-of-planar direction (represented by the 3-axis) [see Fig. 2(b)]. Then, according to the FLDs, this composite is regarded as a combination of laminates, each comprising fibers having the same fiber length [see Fig. 2(c), "L($L_i$), $i = 1, 2, \ldots, n$" denotes the $i$th laminate containing fibers of the same length $L_i$]. Each laminate with the same fiber length is then treated as a stacked sequence of laminae; each lamina consists of fibers having the same fiber length and the same fiber orientation [see Fig. 2(d), "L($L_i$, $\theta_j$), $j = 1, 2, \ldots, m$" denotes the $j$th lamina containing fibers having the same length $L_i$ and the same angle $\theta_j$].

Similar to the prediction of the elastic modulus of SFRP composites,[15] to evaluate the thermal conductivity of the SFRP composite in the "1" direction using the LAA, which depends only on the orientation dis-

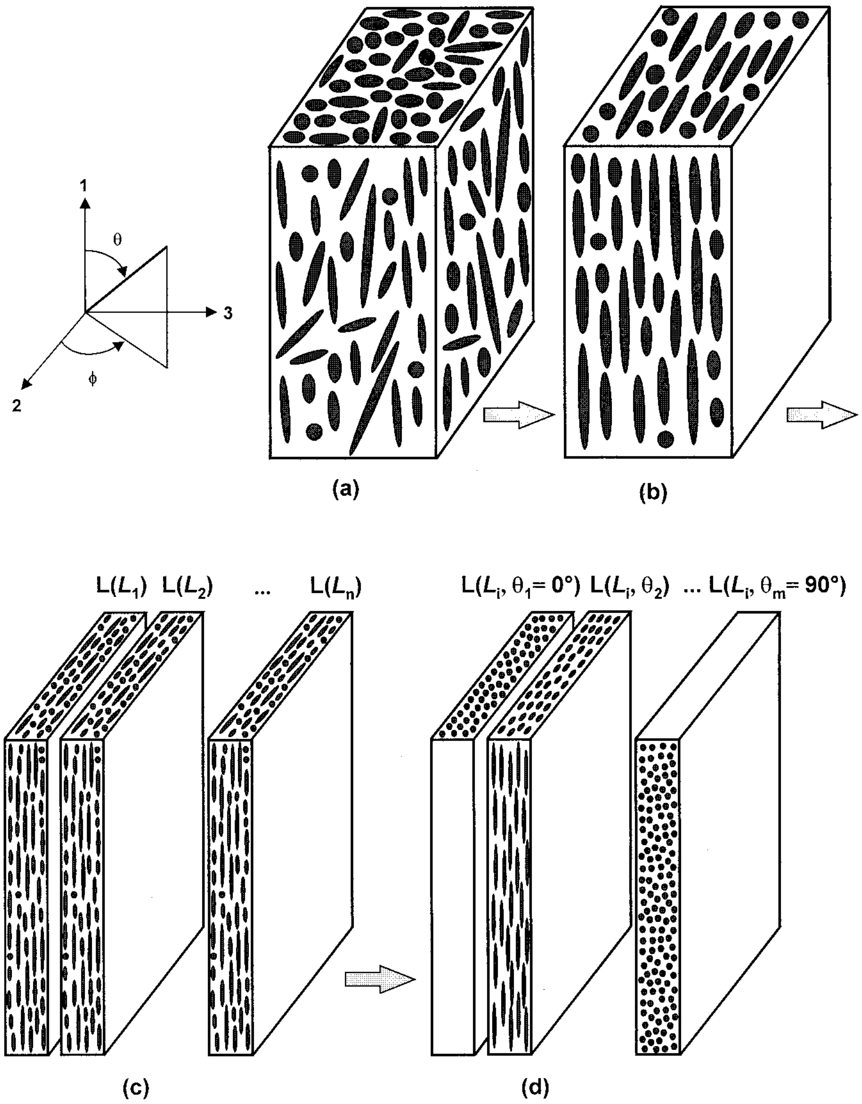



**Figure 2** Simulations of the laminated plate model of a 3D misaligned short-fiber-reinforced polymeric composite: (a) the real 3D SFRP; (b) the supposed SFRP; (c) the supposed SFRP is considered as combination of laminates—each laminate has the same fiber length; and (d) each laminate is treated as a stacked sequence of laminae—each lamina has the same fiber length and the same fiber orientation.

tribution of the angle ($\theta$) that the fibers make with the "1" direction, first we need to evaluate the thermal conductivity of the corresponding unidirectional short fiber composites assuming all the fibers lie in the 1–2 plane. It has been shown[27,28,30,31] that the Halpin–Tsai equation can be used to describe the thermal conduction of unidirectional short fiber composites. For a unidirectional lamina, the thermal conductivities parallel ($K_1$) and perpendicular ($K_2$) to the fiber direction are given by[20]

$$K_1 = \frac{1 + 2\alpha\mu_1 V_f}{1 - \mu_1 V_f} K_m \qquad (9)$$

$$K_2 = \frac{1 + 2\mu_2 V_f}{1 - \mu_2 V_f} K_m \qquad (10)$$

where $\alpha = L/d_f$ in which $d_f$ is the fiber diameter. $V_f$ is the fiber volume fraction. $K_m$ is the thermal conductivity of the matrix. And $\mu_1$ and $\mu_2$ are given by

$$\mu_1 = \frac{K_{f1}/K_m - 1}{K_{f1}/K_m + 2\alpha} \qquad (11)$$

$$\mu_2 = \frac{K_{f2}/K_m - 1}{K_{f2}/K_m + 2} \qquad (12)$$

where $K_{f1}$ and $K_{f2}$ are the thermal conductivity of the fiber in the direction parallel and transverse to the fiber axis direction, respectively.

On the other hand, it has been shown[32] that the thermal conductivity of unidirectional SFRP compos-

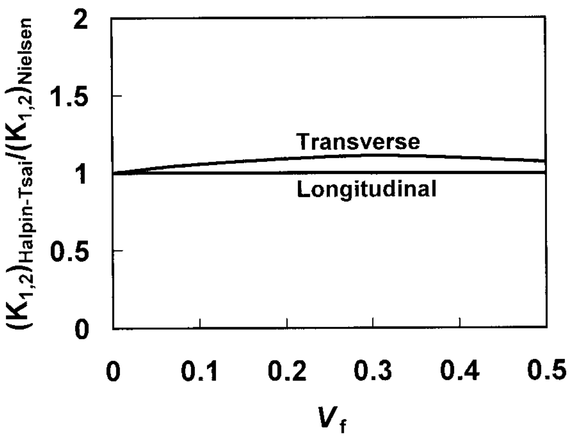



**Figure 3** Thermal conductivity of unidirectional short fiber composites predicted by the Halpin–Tsai equation[20] and the Nielsen equation.[21]

ites can also be predicted accurately by Nielsen's model[21]:

$$K_1 = \frac{1 + 2\alpha\chi_1 V_f}{1 - \chi_1\psi V_f} K_m \tag{13}$$

$$K_2 = \frac{1 + 0.5\chi_2 V_f}{1 - \chi_2\psi V_f} K_m \tag{14}$$

$$\chi_1 = \frac{K_{f1}/K_m - 1}{K_{f1}/K_m + 2\alpha} \tag{15}$$

$$\chi_2 = \frac{K_{f2}/K_m - 1}{K_{f2}/K_m + 0.5} \tag{16}$$

$$\psi = 1 + \left(\frac{1 - V_{\max}}{V_{\max}^2}\right) V_f \tag{17}$$

$V_{\max}$ is maximum fiber fraction possible while still maintaining a continuous matrix phase, and is referred to as the maximum packing fraction. It is easy to ascertain that the Halpin–Tsai and Nielsen equations give similar predictions. Comparison between the two models in prediction of the thermal conductivities of SFRP composites is made in Figure 3, where $V_{\max} = 0.907$ for uniaxial hexagonal alignment ($V_{\max} = 0.785$ for simple cubic alignment, similar results can be obtained), $d_f = 10$ $\mu$m, $L = 0.5$ mm, $K_{f1} = K_{f2} = 10.4$ mW cm$^{-1}$ K$^{-1}$, and $K_m = 2.0$ mW cm$^{-1}$ K$^{-1}$. From Figure 3, it can be seen that the two models give similar (very close) predictions, except there is a small difference in the transverse thermal conductivity. So, the Halpin–Tsai equation will be used in this article for the prediction of thermal conductivity of unidirectional SFRP composites since it does not include the parameter $V_{\max}$, which depends on fiber alignment.

For a unidirectional lamina, the linear relationship between heat flux and temperature gradient in the directions parallel and perpendicular to the fiber direction (namely, in the local coordinate system) is given by

$$q_i = -K_i \nabla T_i \quad i = 1, 2 \tag{18}$$

In the global coordinate system, the fibers of the lamina are oriented at an angle $\theta$ ($\theta \neq 0$) relative to the measured direction; the linear relationship between heat flux and temperature gradient is:

$$q'_i = -K'_i \nabla T'_i \quad i = 1, 2 \tag{19}$$

Let us introduce a transformation tensor $X_{ij}$ defined by

$$Y_i = X_{ij} Y'_j \tag{20}$$

where Yi and $Y'_j$ are the components of a vector $Y$ in the local and global coordinate systems, respectively. Then, we can have

$$X_{ij}^{-1} q'_i = -K_i X_{ij} \nabla T'_j \tag{21}$$

So,

$$K'_i = X_{ij}^{-1} K_i X_{ij} \tag{22}$$

where the coordinate transformation tensor $X_{ij}$ is given by

$$X_{ij} = \begin{Bmatrix} \cos\theta \sin\theta \\ -\sin\theta \cos\theta \end{Bmatrix} \tag{23}$$

$X_{ij}^{-1}$ can be obtained as

$$X_{ij}^{-1} = \begin{Bmatrix} \cos\theta - \sin\theta \\ \sin\theta \; \cos\theta \end{Bmatrix} \tag{24}$$

So we finally obtain

$$K'_1 = K_1 \cos^2\theta + K_2 \sin^2\theta \tag{25}$$

The total heat flux along the 1′ axis in the global coordinate system for a multilaminate is then given by

$$Q'_1 = \sum_{k=1}^{M} q'_1 h_k = -\sum_{k=1}^{M} K'_1 \nabla T'_1 h_k \tag{26}$$

where $M$ represents the number of plies in the laminate, $k$ is the serial index of the ply in the laminate, and $h_k$ is the thickness fraction of the $k$th ply. Since the temperature gradient is continuous across the thickness, eq. (26) is reduced to

$$Q_1' = -K_c \nabla T_1' \quad (27)$$

where the thermal conductivity of the composite lam

$$K_c = \sum_{k=1}^{M} K_1' h_k = \int_{L=L_{\min}}^{L_{\max}} \int_{\theta=\theta_{\min}}^{\theta_{\max}} K_1' f(L) g(\theta) dL d\theta \quad (28)$$

The thermal conductivity of the SFRP composite with a FLD and a FOD can then be evaluated using eq. (28). Nonetheless, if eq. (28) is simply employed to predict composite thermal conductivity, no interaction between short fibers with different orientation angles and different fiber lengths can be considered. To consider the effect of the fiber interaction on the composite thermal conductivity, the fibers with a volume fraction of $V_f$ are divided into two equal halves having the same FLD and FOD. First, the first half of the fibers is incorporated into the pure polymer matrix. The thermal conductivity can then be evaluated using eq. (28). The filled polymer matrix is considered as the effective matrix for the second half of the fibers. It should be noted that the effective fiber volume fraction is $V_f/(2 - V_f)$ and the obtained thermal conductivity is taken as $E_m$ for the second half of the fibers. Second, the second half of the fibers is incorporated into the effective matrix. The interaction between short fibers of different orientation angles and different lengths can then be incarnated in a manner such that the second half of the fibers is incorporated into the effective matrix containing the first half of the fibers. Finally, the composite thermal conductivity can be predicted using eq. (28). And the fiber volume fraction is equal to $V_f/2$.

Moreover, eq. (28) can also be used to evaluate the thermal conductivity of the SFRP composite in any given direction ($\Theta$, $\Phi$), but the angle $\theta$ must be replaced by the angle $\delta$ in eq. (25), where the angle $\delta$ is that between the fiber axial direction ($\theta$,$\phi$) and the direction ($\Theta$, $\Phi$) given by[14]

$$\cos \delta = \cos \Theta \cos \theta + \sin \Theta \cos(\phi - \Phi) \quad (29)$$

The anisotropy of the thermal conductivity of SFRP composites will be reported elsewhere.

## RESULTS AND DISCUSSION

The effect of fiber volume fraction on the thermal conductivity of SFRP composites is shown in Figure 4, where $d_f = 10$ $\mu$m, $K_{f1} = K_{f2} = 10.4$ mW cm$^{-1}$ K$^{-1}$ [for short glass fiber (SGF), see ref. 27], $K_m = 2$ mW cm$^{-1}$ K$^{-1}$ [for poly(phenylene sulfide) (PPS), see ref. 27], $L_{\text{mean}} = 424$ $\mu$m ($a = 2.6$ and $b = 1.2$), and $\theta_{\text{mean}} = 36°$ ($p = 0.6$ and $q = 1$). It is observed that the thermal conductivity of the composites increases almost linearly with increase of fiber volume fraction. This observation is consistent with existing experimental results.[33] This is obvious because the thermal conductivity of fibers is much higher than that of the resin matrix. So, the incorporation of short fibers can significantly enhance the thermal conductivity. Moreover, it can be seen that the composite thermal conductivity after taking into account the effect of the interaction between fibers is higher than that before taking into account this effect. As the fiber volume fraction increases, this effect becomes more significant. This is because the fiber interaction becomes more important as the fiber volume fraction increases. The trend of the variance of the thermal conductivity with other factors remains the same before and after taking into consideration the effect of fiber interaction. In the remainder of this section, for simplicity, only the results before taking into account the effect of fiber interaction will be presented.

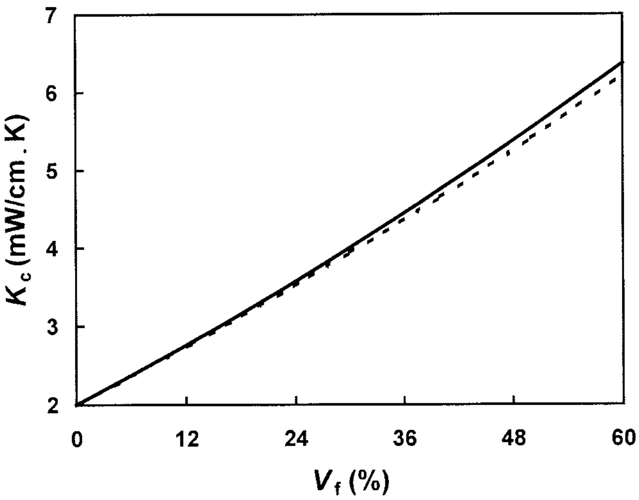



**Figure 4** Effect of fiber volume fraction on thermal conductivity of SFRP composites. Without effect of fiber interaction: dashed line; with effect of fiber interaction: solid line.

Figure 5 shows the effect of mean fiber length (or mean aspect ratio) on the thermal conductivity of SFRP composites, where the parameters are the same as in Figure 4 except $V_f$ is fixed at 0.3, $d_f = 7$ $\mu$m for SCF, the thermal conductivities of carbon fiber are $K_{f1} = 94$ mW cm$^{-1}$K$^{-1}$, $K_{f2} = 6.7$ mW cm$^{-1}$ K$^{-1}$,[27] and $L_{\text{mean}}/d_f$ varies by changing $a$ ($b$ is fixed at 1.2). It can be seen that the thermal conductivity of short-glass fiber-reinforced polymer composites increases very slightly with the increase of mean fiber aspect ratio when $L_{\text{mean}}/d_f$ is small ($<$ about 10) and becomes insensitive to fiber aspect ratio when $L_{\text{mean}}/d_f$ is large ($>$ about 10). However, the thermal conductivity of short carbon-fiber-reinforced polymer composites increases dramatically with the increase of mean fiber aspect ratio especially when $L_{\text{mean}}/d_f <$ about 100. Moreover, the thermal conductivity of the glass fiber

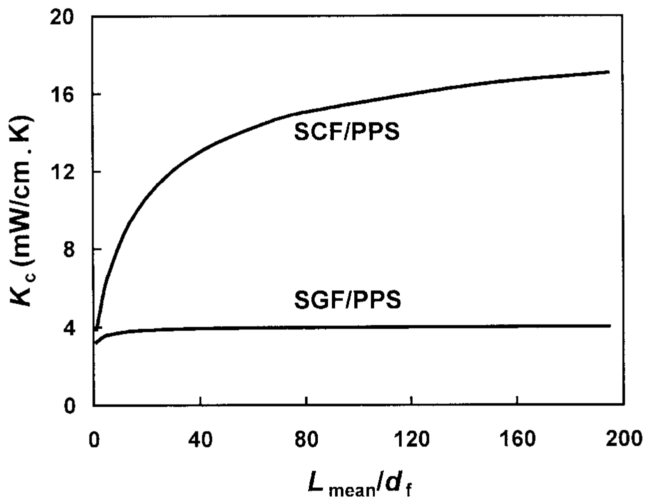



**Figure 5** Effect of mean fiber aspect ratio on thermal conductivity of SFRP composites.

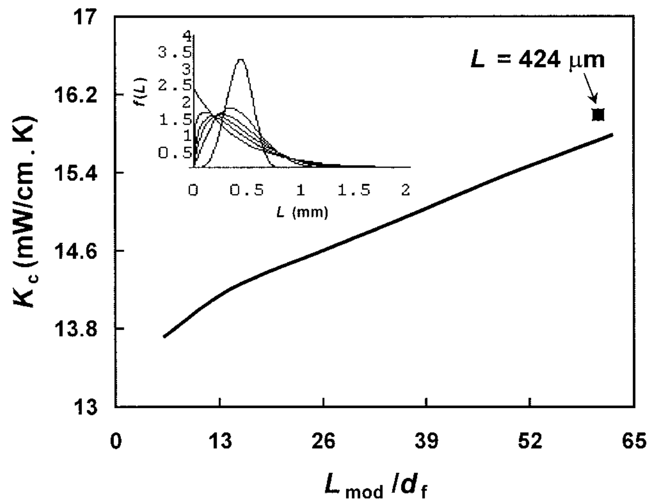



**Figure 7** Effect of mode fiber aspect ratio on thermal conductivity of SFRP composites.

composite is much lower than the thermal conductivity of the carbon fiber composite. This is explained as follows. The thermal conductivity of carbon fiber is much higher than that of glass fiber in the fiber axis direction ($K_{f1}/K_m = 5.2$ for glass fiber and $K_{f1}/K_m = 47$ for carbon fiber), and hence the thermal conductivity of the glass fiber composite will be much lower than that of the carbon fiber composite at a similar fiber content (e.g., by a factor of about 3.8 at $V_f = 0.3$ and $L_{mean}/d_f = 80$) and the thermal conductivity of the glass fiber composite will also be much less sensitive to fiber length than the thermal conductivity of the carbon fiber composite.

Figure 6 displays the effect of mean fiber orientation angle on the thermal conductivity of SFRP composites, where the parameters are the same as in Figure 5 except $L_{mean} = 424$ $\mu$m ($a = 2.6$ and $b = 1.2$) and $\theta_{mean}$ varies by changing $p$ ($q$ is fixed at 1). It can be observed that the thermal conductivity of short-glass fiber-reinforced polymers decreases slowly with increase of mean fiber orientation angle. However, the thermal conductivity of short-carbon fiber-reinforced polymers decreases significantly with increasing mean fiber orientation angle $\theta_{mean}$. This indicates that the thermal conductivity of the carbon fiber composite is more sensitive to fiber orientation than the thermal conductivity of the glass fiber composite. This is because on the one hand, glass fiber is isotropic, and on the other hand, the thermal conductivity of carbon fiber is much higher than that of glass fiber in the fiber axis direction.

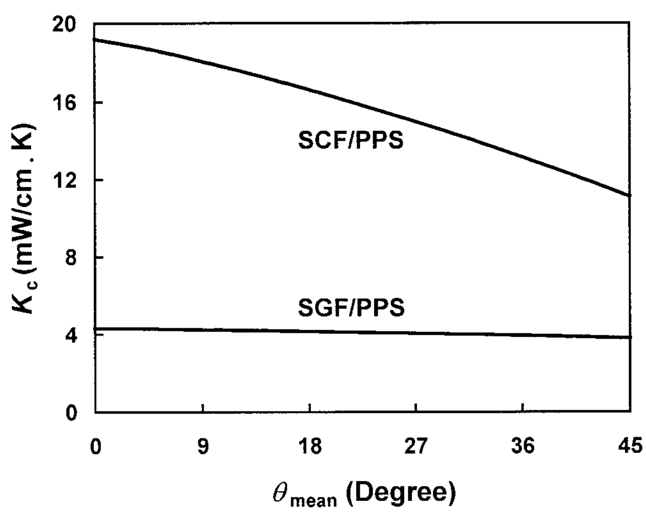



**Figure 6** Effect of mean fiber orientation angle on thermal conductivity of SFRP composites.

Figure 7 shows the effect of mode fiber aspect ratio (or mode fiber length) on the thermal conductivity of SFRP composites, where the parameters are the same as in Figure 6 except $d_f = 7$ $\mu$m only (for carbon fiber/PPS system), $\theta_{mean}$ is fixed at 36° ($p = 0.6$ and $q = 1$) and $L_{mod}$ changes ($L_{mean} = 0.424$ mm). The FLD is also shown in Figure 7. As the mode fiber length increases, the FLD shape changes, i.e., there are fewer fibers of small and large length but more fibers of medium length. It is interesting to note that the composite thermal conductivity increases with the increase of mode fiber length. For the glass fiber/polymer system, similar results can be obtained but the effect becomes smaller because the thermal conductivity of glass fiber is lower. Moreover, it should be pointed out that the thermal conductivity of composites with a constant fiber length is higher than that of a corresponding composite having a FLD (see Fig. 7).

Figure 8 shows the effect of mode fiber orientation angle on the thermal conductivity of SFRP composites, where the parameters are the same as in Figure 7 except $L_{mod}$ is fixed at 101.3 $\mu$m ($a = 2.6$ and $b = 1.2$) and $\theta_{mean}$ is fixed at 25° [$(p,q) = (0.505,2.7)$, $(0.6,3.35)$,

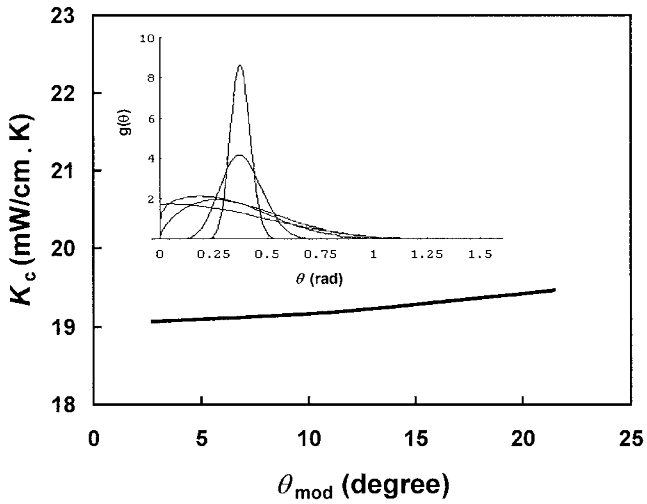



**Figure 8** Effect of mode fiber orientation angle on thermal conductivity of SFRP composites.

(0.8,4.75), (4.0,23.5), (16.0,101)] for the carbon fiber/PPS system. The FOD is also exhibited in Figure 8. The FOD shape changes with the increase of mode fiber orientation angle. Namely, there are more fibers of medium angle and less fibers of small and large angle as the mode fiber orientation angle increases. Figure 8 shows that the composite thermal conductivity increases slowly with the increase of mode fiber orientation angle. Thus, this effect is insignificant. For glass fiber/polymer systems, similar results can be obtained.

## COMPARISON AND APPLICATION

For a two-dimensionally (2D) random short fiber composite, $g(\theta) = 2/\pi$, the expression for the composite thermal conductivity can be obtained form eqs. (25) and (28) for $0 \le \theta < \pi/2$

$$K_c = \frac{1}{2}\left(\int_{L=L_{\min}}^{L_{\max}} K_1 f(L)dL + \int_{L=L_{\min}}^{L_{\max}} K_2 f(L)dL\right) \quad (30)$$

Similarly, for a 3D random short fiber composite, $g(\theta, \phi) = g(\theta)\, g(\phi)/\sin(\theta) = 1/2\pi$, $g(\pi) = 1/2\pi$, and $g(\theta) = \sin\theta$. So, the expression for the composite thermal conductivity can be obtained from eqs. (25) and (28) for $0 \le \theta \le \pi/2$:

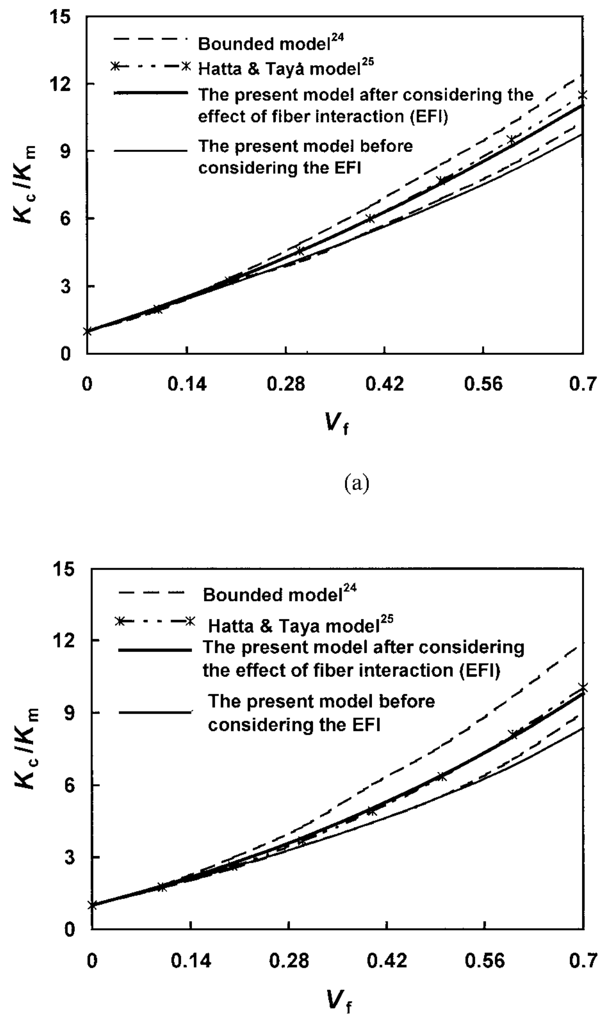
$$K_c = \frac{1}{3}\int_{L=L_{\min}}^{L_{\max}} K_1 f(L)dL + \frac{2}{3}\int_{L=L_{\min}}^{L_{\max}} K_2 f(L)dL \quad (31)$$

When the fiber length has a constant value, eqs. (30) and (31) can be simplified as

$$K_c = \frac{1}{2}(K_1 + K_2) \quad \text{for the 2D random case} \quad (32)$$

$$K_c = \frac{1}{3}K_1 + \frac{2}{3}K_2 \quad \text{for the 3D random case} \quad (33)$$

The above eqs. (32) and (33) are employed to predict the thermal conductivity of 2D and 3D random short fiber composites, respectively. The predicted results are respectively shown in Figures 9(a) and 9(b), where the following values of the parameters are used: $L/d_f = 100$, $K_{f1}/K_m = K_{f2}/K_m = 20$. The predicted results by other theories[24,25] are also displayed in Figure 9. It is seen that the when the fiber interaction is taken into account, the predicted values by the present theory lie between those by the bounded model[24] and are close to those by the Hatta and Taya model,[25] which also




(a)

(b)

**Figure 9** Thermal conductivity of a (a) 2D and (b) 3D random SFRP composite as a function of $V_f$ with $K_{f1}/K_m = K_{f2}/K_m = 20$ and $L/d_f = 100$.

**TABLE I**
**Comparison Between Theoretical Predictions and Experimental Results[27,28] for Thermal Conductivity $K_c$ (mW cm$^{-1}$ K$^{-1}$) of Four Composites**

| Composites | $V_f$ | λ | | $K_c$ | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Surface layer | | | Middle layer | | |
| | | Surface layer | Middle layer | Exp. | Theo.[1 b] | Theo.[2 b] | Exp. | Theo.[1 b] | Theo.[2 b] |
| PEEK30cf[a] | 0.214 | 3.3 | 2.4 | 11.6 | 9.96 | 11.87 | 10.8 | 9.45 | 11.16 |
| PPS30cf | 0.243 | 4.1 | 1.9 | 15.2 | 12.5 | 14.96 | 12.4 | 10.97 | 12.88 |
| PPS40cf[a] | 0.335 | 4.7 | 2.1 | 17.2 | 15.89 | 19.76 | 15.6 | 13.92 | 16.96 |
| PPS40gf[a] | 0.264 | 5.3 | 2.9 | 4.08 | 3.93 | 3.96 | 3.99 | 3.84 | 3.89 |

[a] 30cf, 40cf, and 40gf denote 30 wt % carbon fiber, 40 wt % carbon fiber, and 40 wt % glass fiber, respectively.
[b] Theo.[1] and Theo.[2] denote the theoretical values before and after taking into account the effect of fiber interaction, respectively.

considers the effect of fiber interaction. Although the Hatta and Taya model can give accurate prediction of the thermal conductivity of short fiber composites having a *constant* fiber length, it is not suitable for the prediction of the thermal conductivity of injection-molded SFRP composites having a FLD. When the fiber interaction is not taken consideration, the predicted values by the present model become lower and are close to the lower values predicted by the bounded model.[24]

Moreover, the present theory is applied to published experimental results as shown in Table I for the thermal conductivity of short glass fiber and short carbon fiber reinforced poly(phenylene sulfide) (PPS) composites[27] and poly(ether ether ketone) (PEEK) composites.[28] The thermal conductivities of the surface and skin layers were measured separately. The data for the parameters are as follows:[27,28] $K_m = 2.43$ mW cm$^{-1}$ K$^{-1}$ for PEEK $K_m = 2$ mW cm$^{-1}$ K$^{-1}$ for PPS; $K_1 = K_2 = 10.4$ mW cm$^{-1}$ K$^{-1}$ for glass fiber; $K_1 = 94$ mW cm$^{-1}$ K$^{-1}$ for the carbon fiber used in PPS and $K_1 = 80$ mW cm$^{-1}$ K$^{-1}$ for the carbon fiber used in PEEK and $K_2 = 6.7$ mW cm$^{-1}$ K$^{-1}$. An alternative expression for the FOD density function is given by[27,28]

$$g(\theta) = -\lambda \cdot \exp(-\lambda\theta)/[1 - \exp(-\lambda\pi/2)] \quad (34)$$

and values of λ are given in Table I. The average fiber aspect ratios for the four composites of PEEK30cf, PPS30cf, PPS40cf, and PPS40gf are respectively 17 ($a = 55.4$ and $b = 2$), 21 ($a = 36.3$ and $b = 2$), 16 ($a = 62.7$ and $b = 2$), and 17 ($a = 27$ and $b = 2$).[27,28] Clearly, the theoretical results are in good agreement with the experimental values when the effect of the interaction between fibers is taken into account. When the effect of fiber interaction is not included, the predicted results are lower than the experimental values.

## CONCLUSIONS

The effects of fiber volume fraction, fiber length, and orientation distributions on the thermal conductivity of SFRP composites have been studied. The composite thermal conductivity increases almost linearly with increase of fiber volume fraction. When the fiber thermal conductivity is high, the composite thermal conductivity increases significantly with the increase of mean fiber aspect ratio (or mean fiber length) and decrease of mean fiber orientation angle. When the fiber thermal conductivity is low, the composite thermal conductivity increases very slowly with the increase of mean fiber aspect ratio (or mean fiber length) and decrease of mean fiber orientation angle. The effects of mode fiber length and mode fiber orientation on the thermal conductivity of SCF/PPS composites have been studied, and the results show that the composite thermal conductivity increases marginally with the increase of mode fiber aspect ratio (or mode fiber length) but slightly with increase of mode fiber orientation angle. Moreover, the theoretical predictions after taking into account the effect of fiber interaction agree well with published experimental data.

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